Math 253
Spring 2021

**Name:** Key

**Quiz 6.1**

1. (10 points) Find the center, radius of convergence, and interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{3^n}{n^2} \cdot (x+1)^n$$

$$\lim_{n\to\infty} \left| \frac{3^{n+1}(x+1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{3^n \cdot (x+1)^n} \right| = |x+1| \cdot \lim_{n\to\infty} \left| 3 \cdot \frac{n^2}{(n+1)^2} \right|$$

$$= |x+1| \cdot 3$$

$$3|x+1| < 1 \iff |x+1| < \frac{1}{3} \iff x \in \left(-\frac{4}{3}, -\frac{2}{3}\right)$$

Check: $x = -\frac{2}{3} \to \sum_{n=0}^{\infty} \frac{3^n}{n^2} \cdot \left(-\frac{2}{3}+1\right)^n = \sum_{n=0}^{\infty} \frac{3^n}{n^2} \cdot \left(\frac{1}{3}\right)^n$

$$= \sum_{n=0}^{\infty} \frac{1}{n^2}$$ which converges by the p-test ($p=2$)

$$x = \frac{-4}{3} \to \sum_{n=0}^{\infty} \frac{3^n}{n^2} \cdot \left(-\frac{4}{3}+1\right)^n = \sum_{n=0}^{\infty} \frac{3^n}{n^2} \left(-\frac{1}{3}\right)^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2}$$ converges by the A.S.T.

$\Rightarrow$ Center: $x = -1$

$R = \frac{1}{3}$

$I = \left[-\frac{4}{3}, -\frac{2}{3}\right]$